18.1.4 – 18.1.6 CALCULATIONS INVOLVING ACIDS AND BASES

Review of Important formulas

$pH = -\log_{10}[H^+]$ $[H^+] = 10^{-pH}$

$pK_a = -\log_{10} K_a$ $K_a = 10^{-pKa}$

$pOH = -\log_{10}[OH^-]$ $[OH^-] = 10^{-pOH}$

$pK_b = -\log_{10} K_b$ $K_b = 10^{-pKb}$

The ionic product of water = $K_w = [H^+] \times [OH^-] = 1.0 \times 10^{-14}$ $mol^2\ dm^{-6}$ at 298 K
The expression varies with temperature

$pH + pOH = 14$

$K_a \times K_b = 1.0 \times 10^{-14}$ $pK_a + pK_b = 14$

Acids

The K_a is the acid dissociation constant and is a measure of the strength of an acid or in other words a measure of the ability of an acid to dissociate into ions

Complete the table and statements below

K_a	pK_a
1×10^{-3}	
	9.4
1×10^{-5}	
	9.2

- Compared to a weak acid, a strong acid will contain a __________ concentration of H^+ ions, have a __________ K_a, a __________ pK_a and according to Bronsted-Lowry theory a __________ conjugate base.
- In comparison, a weak acid will contain a lower concentration of __________ ions, have a small __________, a large pK_a and a __________ conjugate base.

- When the strength of the acid decreases the K_a __________ and the pK_a __________.

The following generalized equation is often used to show the dissociation of an acid in water:

$HA_{(aq)}$	$\Leftrightarrow$	$H^+_{(aq)}$	+	$A^-_{(aq)}$
general acid				conjugate base

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

Note: that water is not included as one of the reactants in the equation because it is a pure liquid and so its concentration cannot be determined.

1. Strong Acids

- A strong acid completely dissociates into ions. Therefore we make the approximation (assumption) that the concentration of the acid molecules is equal to the concentration of the hydrogen ions, because all of the acid molecules are converted into H+ and A- ions.
- This approximation need to be stated when calculating the pH of a strong acid.

Example:
Calculate the pH and hydroxide ion concentration of a strong as with a $[H^+] = 1 \times 10^{-2}$ mol dm^{-3}.

Answer:
Approximation: A strong acid therefore $[HA] = [H^+]$

$HA_{(aq)}$	$\Leftrightarrow$	$H^+_{(aq)}$	+	$A^-_{(aq)}$
1×10^{-2} mol dm^{-3}		1×10^{-2} mol dm^{-3}		1×10^{-2} mol dm^{-3}

$pH = -\log_{10}[H^+] = -\log_{10} 1 \times 10^{-2} = 2$ *(1SF)*

$$[OH^-] = \frac{1 \times 10^{-14}}{[H^+]} = \frac{1 \times 10^{-14}}{1 \times 10^{-2}} = 1 \times 10^{-12} \text{ mol dm}^{-3} \text{ (1SF)}$$

2. Weak Acids

- A weak acid does not completely dissociate into ions. Therefore, at equilibrium there are very few moles of ions, the solution contains mostly undissociated acid molecules.
- In calculations involving a weak acid the following approximations are made in order to simplify the model. These approximations need to be stated during calculations.

Approximations

1. $[H^+_{(aq)}] = [A^-_{(aq)}]$ and both have a concentration of χ mol L^{-1}
2. that the concentration of $[HA_{(aq)}] = [HA_{(aq)}] - \chi$, but because the concentration of χ is very small we assume it is negligible so $[HA_{(aq)}] = [HA_{(aq)}]$

$$HA_{(aq)} \Leftrightarrow H^+_{(aq)} + A^-_{(aq)}$$

$$\qquad\qquad\qquad \chi \qquad\quad \chi$$

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$K_a = \frac{\chi \times \chi}{[HA_{(aq)}]}$$

Example 1

Find the $[H^+_{(aq)}]$ and pH of a 0.0200 mol dm^{-3} solution of a weak acid with a $K_a = 6.17 \times 10^{-8}$ mol dm^{-3}.

Answer

Approximations

1. $[H^+_{(aq)}] = [A^-_{(aq)}] = \chi$ *mol* dm^{-3}
2. $[HA_{(aq)}] = [HA_{(aq)}] - \chi$, *but because the concentration of* χ *is very small assume* $[HA_{(aq)}] = [HA_{(aq)}]$

$HA_{(aq)}$	$\Leftrightarrow$	$H^+_{(aq)}$	+	$A^-_{(aq)}$
0.0200 mol dm^{-3}		χ		χ

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$6.17 \times 10^{-8} = \chi \times \chi$$

$$\frac{}{[HA_{(aq)}]}$$

$$6.17 \times 10^{-8} = \frac{\chi^2}{0.0200}$$

$$\chi^2 = 6.17 \times 10^{-8} \times 0.0200$$

$$[H^+_{(aq)}] = 3.51 \times 10^{-5} \text{ mol dm}^{-3} \text{ (3SF)}$$

$[H^+_{(aq)}] = 3.51 \times 10^{-5}$ mol dm^{-3}

so $pH = -\log_{10}[H^+] = -\log_{10} 3.51 \times 10^{-5} = 4.45$ (3SF)

Example 2
A 0.010 moldm^{-3} solution of a weak acid has a pH of 4. Determine the K_a of the acid.

Answer

Bases

- **K_b** is the base dissociation constant and is a measure of the strength of an base, or in other words a measure of the ability of the base to dissociate into ions, one of which will be a hydroxide ion (Arrhenius theory)
- **pK_b** = $-\log_{10} K_b$
- A strong base will contain a higher concentration of OH^- ions, have a large K_b, a small pK_b and weaker conjugate acid.
- A weak base will contain a lower concentration of OH^- ions, have a small K_b, a larger pK_b and stronger conjugate acid.

K_a, pK_a, K_b and pK_b can be related using the equations:

$$K_a \times K_b = 1 \times 10^{-14}$$

$$pK_a + pK_b = 14$$

To write an equation for the dissociation of a base in water the following generalized equation is often used.

$$B_{(aq)} + H_2O \Leftrightarrow BH^+_{(aq)} + OH^-_{(aq)}$$

base — conjugate acid

$$K_b = \frac{[BH^+_{(aq)}] \times [OH^-_{(aq)}]}{[B_{(aq)}]}$$

Note: that water is included as one of the reactants in the equation but is not included in the K_b.

The same method and approximations used to calculate the concentration of strong and weak acids is used for strong and weak bases except that K_b is used instead of K_a and the $[OH^-_{(aq)}]$ has to be determined before finding the pH.

Example 3

The pK_b for a base is 5.0. Determine the pH of a 0.10 moldm^{-3} solution of the base.

Answer

$K_b = 10^{-pKb} = 10^{-5.0} = 1.0 \times 10^{-5}$

Approximations

1. $[OH^-_{(aq)}] = [BH^+_{(aq)}] = \chi \ mol\ dm^{-3}$

2. $[B_{(aq)}] = [B_{(aq)}] - \chi$, *but because the concentration of* χ *is very small assume* $[B_{(aq)}] = [B_{(aq)}]$

$$B_{(aq)} + H_2O \Leftrightarrow BH^+_{(aq)} + OH^-_{(aq)}$$

base *conjugate acid*

$$K_b = \frac{[BH^+_{(aq)}] \times [OH^-_{(aq)}]}{[B_{(aq)}]}$$

$$1.0 \times 10^{-5} = \frac{\chi \times \chi}{[B_{(aq)}]}$$

$$1.0 \times 10^{-5} = \frac{\chi^2}{0.10}$$

$$\chi^2 = 1.0 \times 10^{-5} \times 0.10$$

$$[OH^-_{(aq)}] = 0.0010 \text{ mol dm}^{-3} \text{ (2SF)}$$

$[OH^-_{(aq)}] = 0.0010 \text{ mol dm}^{-3}$

$pOH = -\log_{10}[OH^-] = -\log_{10} 0.0010 = 3.0$

$pH + pOH = 14$

$pH = 14 - 3.0 = 11$

18.1 Questions

In order to receive full or partial credit for these questions the method used and all the steps involved at arriving at your final answer must be shown clearly. Please show all units and use the correct number of significant figures. For multiple choice questions please don't use a calculator.

1. (N00/H) What is the Ka of a 0.10 moldm^{-3} solution of a weak monoprotic acid if the $[H^+] = 2.0 \times 10^{-3}$ mol dm^{-3}? *Don't use a calculator for this question*.
   A. 2.0×10^{-2} mol dm^{-3}
   B. 2.0×10^{-4} mol dm^{-3}
   C. 4.0×10^{-5} mol dm^{-3}
   D. 4.0×10^{-7} mol dm^{-3}

2. (N04/H) The acid dissociation constant of a weak acid HA has a value of 1.0×10^{-5} $moldm^{-3}$. What is the pH of a 0.10 $moldm^{-3}$ aqueous solution of HA?
Don't use a calculator for this question.
   A. 2
   B. 3
   C. 5
   D. 6

3. (M03/H) The K_a value for an acid is 1.0×10^{-2}. What is the K_b value for its conjugate base? *Don't use a calculator for this question*.
   A. 1.0×10^{-2}
   B. 1.0×10^{-6}
   C. 1.0×10^{-10}
   D. 1.0×10^{-12}

4. (N01/H) A 0.1 mol dm^{-3} solution of a weak acid has a pH = 3.0. What is the K_a for this acid? *Don't use a calculator for this question*.
   A. 1.0×10^{-1}
   B. 1.0×10^{-3}
   C. 1.0×10^{-5}
   D. 1.0×10^{-6}

5. (N01/H) The acid HA has the dissociation constant, Ka, in aqueous solution. What is the equilibrium constant for the reaction below? *Don't use a calculator for this question*.

$$A^-_{(aq)} + H_2O_{(l)} \Leftrightarrow HA_{(aq)} + OH^-_{(aq)}$$

   A. $\frac{K_w}{K_a}$

   B. $\frac{K_a}{K_w}$

   C. K_a

   D. $\frac{1}{K_a}$

6. Hydrochloric acid is a strong acid. In a 0.010 $moldm^{-3}$ solution, calculate:
   a) pH
   b) hydroxide ion concentration

7. Some weak acids and their pK_a values are shown below. Which one of these acids will have the strongest conjugate base? Explain.

Acid	**pK_a**
methanoic	3.75
bromothanoic	2.90
phenol	10.00
methylpropanoic	4.85

8. The K_a for 2-nitrophenol is 6.17×10^{-8}. Calculate:
   a) The pK_a
   a) The pH of a 0.010 $moldm^{-3}$ solution

9. A 0.29 $moldm^{-3}$ solution of a weak acid has a pH of 4.76.
   a) Calculate the K_a for the acid.
   b) Is it a stronger or weaker acid than ethanoic acid (pk_a=4.76)

10. (M05/H) Determine the pOH and pH of a solution with an ammonia concentration of 0.121 $moldm^{-3}$ (pK_b of ammonia is 4.75) *[5]*

11. (M04/H/1) What is the relationship between K_a and pk_a?

12. (N03/H)
   a) Calculate the K_a value of methanoic acid, HCOOH, using table 15 in the Data Booklet. *[1]*
   b) Based on its K_a value, state and explain whether methanoic acid is a strong or weak acid. *[2]*
   c) Calculate the hydrogen ion concentration and the pH of a 0.010 $moldm^{-3}$ methanoic acid solution. State one assumption made in arriving at your answer. *[4]*

13. (M02/H) What are the [H^+(aq)] and [OH^-(aq)] in a 0.1 $moldm^{-3}$ solution of a weak acid ($K_a = 1.0 \times 10^{-7}$)? *Don't use a calculator for this question*.

	$[H^+]$ mol dm^{-3}	$[OH^-]$ mol dm^{-3}
A.	1.0×10^{-1}	1.0×10^{-13}
B.	1.0×10^{-3}	1.0×10^{-11}
C.	1.0×10^{-4}	1.0×10^{-10}
D.	1.0×10^{-6}	1.0×10^{-8}

14. (M02/H)
   a) Calculate the pH value of a 0.1 mol dm^{-3} hydrochloric acid, and suggest a value for 0.1 mol dm^{-3} ethanoic acid. [2]
   b) Calculate the K_a value of ethanoic acid, using table 16 in the Data Booklet. [1]
   c) Write an expression for the ionisation constant, K_a for ethanoic acid. [1]
   d) Calculate the pH of a 0.050 mol dm^{-3} solution of ethanoic acid. [2]

15. N98/H(1)
The K_a for a weak monoprotic acid is 1 x 10^{-5} mol dm^{-3}. What will be the pH of a solution of this acid with a concentration of 0.1 mol dm^{-3}?

Answer to Example 2

A 0.010 $moldm^{-3}$ solution of a weak acid has a pH of 4. Determine the K_a of the acid.

Approximations

1) $[H^+_{(aq)}] = [A^-_{(aq)}]$
2) $[HA_{(aq)}] = [HA_{(aq)}] - \chi$, but because the concentration of χ is very small assume $[HA_{(aq)}] = [HA_{(aq)}]$

$HA_{(aq)}$	$\Leftrightarrow$	$H^+_{(aq)}$	+	$A^-_{(aq)}$
0.010 mol dm^{-3}		χ		χ

$[H^+] = 10^{-pH} = 10^{-4} = 1 \times 10^{-4}$ $moldm^{-3}$

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$= \frac{1 \times 10^{-4} \times 1 \times 10^{-4}}{0.010}$$

$$= 1 \times 10^{-6} \text{ mol dm}^{-3} \text{ (1SF)}$$

18.3 Calculations of acids and bases **Answers**

1. (N00/H) C

2. (N04/H) B

3. (M03/H) D

4. (N01/H) C

Weak acid approximations

pH = 3 so $[H^+_{(aq)}] = 1 \times 10^{-3}$

that $[H^+_{(aq)}] = [A^-_{(aq)}] = 1 \times 10^{-3}$ mol dm^{-3}

The concentration of [$HA_{(aq)}$] = [$HA_{(aq)}$] - χ , but because the concentration of χ is very small assume [$HA_{(aq)}$] = [$HA_{(aq)}$] = 0.1 mol dm^{-3}

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$= \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{0.1}$$

$$= \frac{1 \times 10^{-6}}{0.1}$$

$$= 1 \times 10^{-5} \text{ mol dm}^{-3}$$

5. (N01/H) A

$$K_a = \frac{[H^+] \times [A^-]}{[HA_{(aq)}]}$$

$$[HA] = \frac{[H+] \times [A^-]}{K_a}$$

$$K_c = \frac{[HA] \times [OH^-]}{[A^-]}$$

$$K_c = \frac{[H+] \times [A^-] \times [OH^-]}{[A^-] \times K_a}$$

$$K_c = \frac{[H+] \times \cancel{[A^-]} \times [OH^-]}{\cancel{[A^-]} \times K_a}$$

$$K_c = \frac{K_w}{K_a}$$

6.

a) pH = - $\log_{10}$ [H^+] = - $\log_{10}$ 0.010 = 2

b) pH + pOH = 14

pOH = 14 = 2 = 12

[OH^-] = 10^{-pOH} = 1 x 10^{-12} moldm^{-3}

7.

Acid	pK_a	K_a
methanoic	3.75	1.78×10^{-4}
bromothanoic	2.90	0.00126
phenol	10.00	1.00×10^{-10}
methylpropanoic	4.85	1.41×10^{-5}

The weakest acid has the smallest Ka and largest pKa and according to B-L theory will have the strongest conjugate base.

8. a) $pK_a = -\log_{10}[K_a] = -\log_{10} 6.17 \times 10^{-8} = 7.21$ (3SF)

b)

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$6.17 \times 10^{-8} = \frac{\chi \times \chi}{[HA_{(aq)}]}$$

$$6.17 \times 10^{-8} = \frac{\chi^2}{0.010}$$

$$\chi^2 = 6.17 \times 10^{-8} \times 0.010$$

$$[H^+_{(aq)}] = 2.4 \times 10^{-5} \text{ mol dm}^{-3}$$

(2SF)

$[H^+_{(aq)}] = 2.4 \times 10^{-5}$ mol dm^{-3}
pH = $-\log_{10}[H^+] = -\log_{10} 2.4 \times 10^{-5} = 4.6$

9. A 0.29 moldm^{-3} solution of a weak acid has a pH of 4.76.
   c) Calculate the K_a for the acid.
   d) Is it a stronger or weaker acid than ethanoic acid (pk_a=4.76)

Approximations

1. $[H^+_{(aq)}] = [A^-_{(aq)}]$ and that both have a concentration
2. concentration of $[HA_{(aq)}] = [HA_{(aq)}] - \chi$, but because the concentration of χ is very small assume $[HA_{(aq)}] = [HA_{(aq)}]$

$$HA_{(aq)} \Leftrightarrow H^+_{(aq)} + A^-_{(aq)}$$

0.29
mol dm^{-3}

$[H^+] = 10^{-pH} = 10^{-4.76} = 1.74 \times 10^{-5}$ moldm^{-3}

$$K_a = \frac{[H^+_{(aq)}] \quad x \quad [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$= \frac{1.74 \times 10^{-5} \quad x \quad 1.74 \times 10^{-5}}{0.29}$$

$$= 1.04 \times 10^{-9} \text{ mol dm}^{-3}$$

b) pKa $= -\log_{10} K_a = 8.98$

The strongest acid will have the smallest Ka and the largest pKa. This acid is a weaker than ethanoic acid which has a pk_a=4.76.

10. (M05/H)

$K_b = 10^{-pKb} = 10^{-4.75} = 1.78 \times 10^{-5}$

Assumptions

1. $[OH^-_{(aq)}] = [BH^+_{(aq)}]$ and that both have a concentration = χ mol dm^{-3}
2. concentration of $[B_{(aq)}] = [B_{(aq)}] - \chi$, but because the concentration of χ is very small assume $[B_{(aq)}] = [B_{(aq)}]$

$$NH_{3(aq)} + H_2O \Leftrightarrow NH_4^+{}_{(aq)} + OH^-_{(aq)}$$

base — conjugate acid

$$K_b = \frac{[NH_4^+{}_{(aq)}] \quad x \quad [OH^-_{(aq)}]}{[NH_{3(aq)}]}$$

$$1.78 \times 10^{-5} = \frac{\chi \quad x \quad \chi}{[NH_{3(aq)}]}$$

$$1.0 \times 10^{-5} = \frac{\chi^2}{0.121}$$

$$\chi^2 = 1.78 \times 10^{-5} \quad x \quad 0.121$$

$$[OH^-_{(aq)}] = 0.00146 \text{ mol dm}^{-3} \text{ (3SF)}$$

$[OH^-_{(aq)}] = 0.00146 \text{ mol dm}^{-3}$

$pOH = -\log_{10}[OH^-] = -\log_{10} 0.00146 = 2.84$ (3SF)

$pH + pOH = 14$

$pH = 14 - 2.84 = 11.2$ (3SF)

11. (M04/H)

$pK_a = -\log_{10} K_a$ in the same way that $pH = -\log_{10}[H^+]$)

12.

a) $K_a = 10^{-3.75} = 1.78 \times 10\text{-}^{4} \text{ mol dm}^{-3}$;

b) Weak ;
Ka <1;

c) Weak acid approximations;
$[H^+_{(aq)}] = [A^-_{(aq)}] = x \text{ mol dm}^{-3}$
Concentration of $[HA_{(aq)}] = [HA_{(aq)}] - \chi$, but because the concentration of χ is very small assume $[HA_{(aq)}] = [HA_{(aq)}] = 0.1 \text{ mol dm}^{-3}$

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$1.78 \times 10^{-4} = \frac{\chi \times \chi}{[0.010_{(aq)}]}$$

$$\chi^2 = 1.78 \times 10^{-4} \times 0.010$$

$$\sqrt{\chi^2} \quad 0.0013 \text{ mol}^2\text{dm}^{-6}$$

$$[H^+_{(aq)}] = 0.0013 \text{ mol dm}^{-3} \;;$$

$$pH = 2.9 \;;$$

13. (M02/H) C

$$K_a = \frac{[H^+_{(aq)}] \times [A^-_{(aq)}]}{[HA_{(aq)}]}$$

$$1.0 \times 10^{-7} = \frac{\chi \times \chi}{[HA_{(aq)}]}$$

$$1.0 \times 10^{-7} = \frac{\chi^2}{0.1}$$

$$\chi^2 = 1.0 \times 10^{-7} \times 0.1$$

$$= 1.0 \times 10^{-8}$$

$$\sqrt{\chi^2} \quad 1.0 \times 10^{-4}$$

$$[H^+_{(aq)}] = 1.0 \times 10^{-4} \text{ mol dm}^{-3}$$

$$[OH^-_{(aq)}] = 1.0 \times 10^{-10} \text{ mol dm}^{-3}$$

14.

a) HCl is a strong acid so $[HA_{(aq)}] = [H+_{(aq)}]$ pH = 1 ;
ethanoic acid of the same concentration has pH = 3 ;

b) $K_a = 10^{-4.76} = 1.74 \times 10\text{-}^{5}$ mol dm^{-3};

c)

$$K_a = \frac{[H^+_{(aq)}] \times [CH_3COO^-_{(aq)}]}{[CH_3COOH_{(aq)}]}$$

d)

$$\chi^2 = 1.74 \times 10^{-5} \times 0.050$$

$$[H^+_{(aq)}] = 9.3 \times 10^{-4} \text{ mol dm}^{-3} \text{ ;}$$

$$pH = 3.0 \text{ ;}$$

15.

$$\chi^2 = 1 \times 10^{-5} \times 0.1$$

$$[H^+_{(aq)}] = 0.001 \text{ mol dm}^{-3}$$

$$pH = 3 \text{ ;}$$